Un Steady state Conduction
[Transient conduction]

→ when the heat Energy is being added or removed to or from a body, its Energy content (Internal Energy) changes, resulting into change in its temparature at each point within the body over the time.

→ In unsteady state conduction Temparature varies with time

$\frac{dT}{dt} \neq 0$.

→ During this ~~period~~ transient period, the temparature becomes function of time as well as direction in the body. The conduction occured ~~this~~ during this period is called Transient conduction.

→ The temparature and Heat ~~cond~~ conduction are depends both on the time ~~and~~ space cooldinates, ie $T = f(x, y, z, t)$

→ In many engineering applications, the heat transfered is transient. The heat treatment process like Quenching, Annealing, normalizing etc --- are process of unsteady state heat flow.

→ change in temparature during unsteady state may follow: a periodic (or) a non-periodic variation.

→ The temparature changes in repeated cycles and the conditions get repeated after some fixed time inveral is called periodic variation.

→ Temparature variations in the cylinder of ~~IC~~ an I-C Engine are considered periodic with respect to ~~the~~ Crank angle

Ex:- periodic heat flow in a building between day and NIGHT.

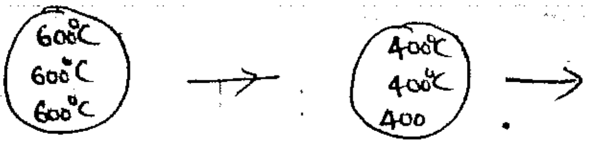
→ The temperature changes as some non-linear function of time. This variation is neither according to definite pattern nor is repeated cycles. Is called non-periodic variation.

Ex:- heating of an ingot in furnace, cooling of bars.

Lumped system Analysis :- ①

In lumped system Analysis temperature varies with time, but not with space ie, a lumped system analysis temp is the function of time only.

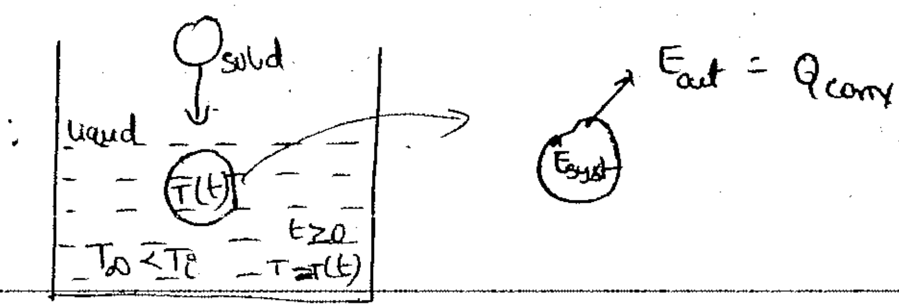
Ex:- A Copper ball taken from oven



→ lumped system is valid

→ lumped system is invalid

Equation for time taken by an object to reach a particular temperature :-



solid suddenly exposed to convection environment T_∞.

Let 'T' be Temp of the object at any temperature 't'

T_i be the temp of initial temp of the object

A be the surface area of the object

h be the heat transfer co-efficient

Heat convected from the surface = Energy stored in the object

Heat convected = $hA(T_\infty - T)\,dt$

Energy stored = $mc_p\,dT$

↳ m = mass of object

c_p = spl heat of the object

$\therefore\ hA(T_\infty - T)\,dt = mc\,dT$ $\qquad \rho = \frac{m}{V}$

$hA(T_\infty - T)\,dt = \rho Vc\,dT.$ $\qquad m = \rho V.$

$hA(T_\infty - T)\,dt = \rho cV \cdot d(T)$

$-hA(T - T_\infty)\,dt = \rho cV \cdot d(T)$

$$\int_0^t -\frac{hA}{\rho cV}\cdot dt = \int_{T_i}^{T} \frac{d(T)}{(T - T_\infty)}$$

$$-\frac{hA}{\rho cV}\int_0^t dt = \int_{T_i}^{T} \frac{d(T)}{T - T_\infty}$$

$$-\frac{hA}{\rho cV}[t]_0^t = \ln(T - T_\infty)\Big|_{T_i}^{T}$$

$$-\frac{hA}{\rho C V} t = \ln\left[\frac{T - T_\infty}{T_i^0 - T_\infty}\right] \quad (3)$$

$$\boxed{\frac{T - T_\infty}{T_i^0 - T_\infty} = e^{-\frac{hA}{\rho C V} t}}$$

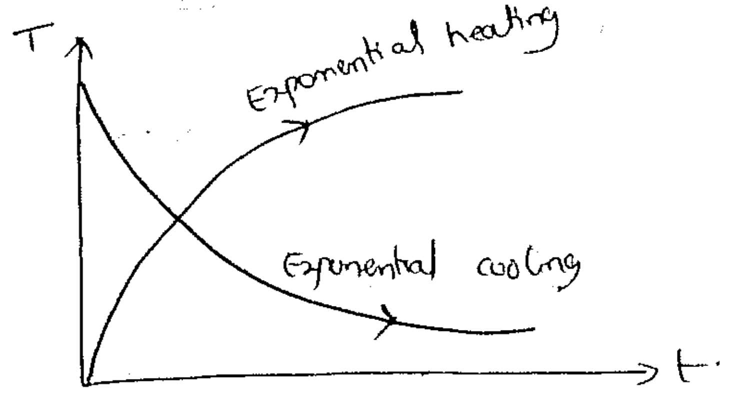
Observation of Above Equation :-

1) The equation can be used to determine the time t required for solid to reach some Temparature T

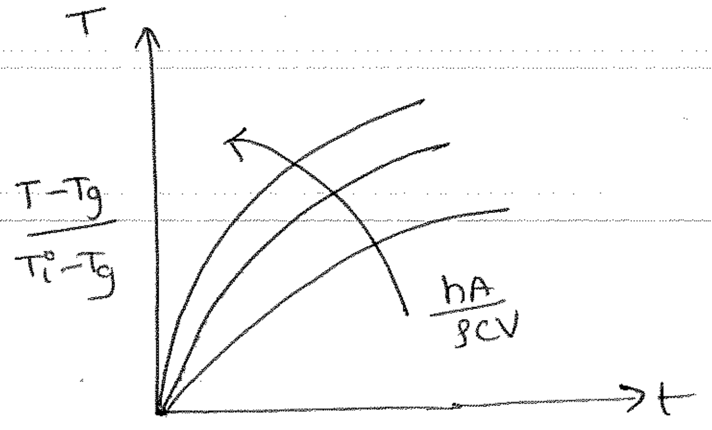
2) The temp of a body approaches the ambient temp T_∞ exponentially. The temp of the body changes rapidly at the begining [due to large temp difference], but slow down later on.

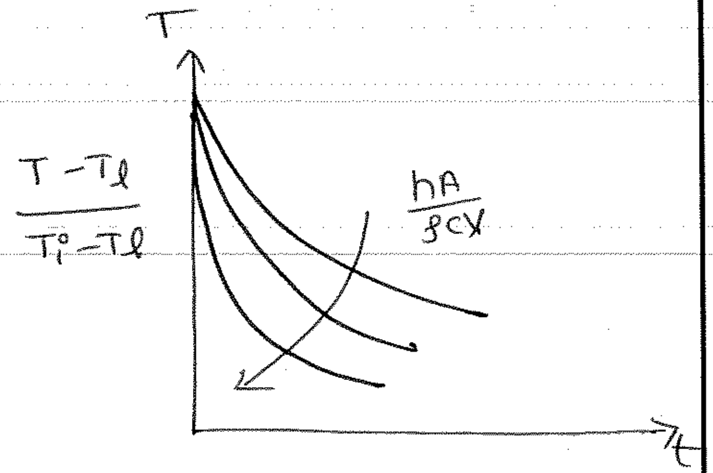
3) The large value of $\frac{hA}{\rho C V}$ indicates that the body will approach the ambient temp in a short time

4) The temparature varies with time exponentially.



Heating and cooling of body when $K \to \infty$





T_g - Gas temp > Body temp

T_l - liquid temp < Body temp

→ ~~...~~

Biot Number :-

→ In General accepted that lumped system analysis is applicable if

$$B_i \leq 0.1$$

→ When $B_i < 0.1$, the variation of temp with location within the body is slight and can reasonably be approximated as being uniform.

→ The first step in the application of lumped system analysis is the calculation of Biot number

→ If the Biot number (B_i) is Less than 0.1 we should approach lumped system analysis, otherwise we should following graphical method of ~~Heist~~ Heisler charts and Grober charts

→ Biot Number is the ratio of the conductive resistance to the convective resistance

$$\frac{R_{cond}}{R_{conv}} = \frac{\frac{L}{KA}}{\frac{1}{hA}} = \frac{L}{K\cancel{A}} \times \frac{h\cancel{A}}{1} = \frac{hL}{K}$$

(OR)

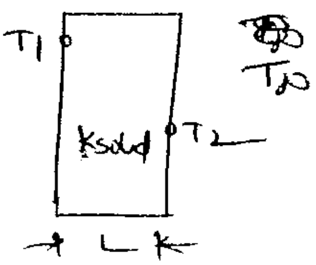
→ Ratio of the convection at the surface to conduction with in the body.

$$\frac{Q_{conv}}{Q_{cond}} = \frac{hA\,dT}{KA\frac{dT}{dx}} = \frac{h\,dx}{AK} = \frac{hL}{K.}$$

(OR)

→ It is the ratio of temp drop due to conduction to the temp drop due to convection

$$\frac{KA(T_1-T_2)}{L} = hA(T_2-T_\infty)$$



$$\frac{T_1-T_\infty}{T_2-T_\infty} = \frac{hAL}{KA} = \frac{\frac{L}{KA}}{\frac{1}{hA}}$$

$$\therefore = \frac{\text{conductive resistance}}{\text{convective Resistance}} = B_i$$

$$\therefore \frac{T_1 - T_2}{T_2 - T_\infty} = \frac{hL}{K_{solid}} = B_i$$

$$\Rightarrow \boxed{B_i = \frac{T_1 - T_2}{T_2 - T_\infty} = \frac{hL}{K_{solid}} = \frac{\Delta T_{cond}}{\Delta T_{conv}} = \frac{R_{cond}}{R_{conv}}}$$

$$\boxed{Bi = \frac{ICR}{ECR}}$$

ICR → Internal conductive Resistance
L/KA

ECR → external conductive Resistance

$$\cancel{ICR} = \frac{\cancel{L}}{\cancel{K}}$$

$$\uparrow \frac{L}{KA} = I.C.R \downarrow = B$$

$$B \downarrow \frac{I.C.R}{ECR} = Bi \downarrow \quad \& \quad \uparrow \frac{I.C.R}{ECR} = Bi \uparrow$$

→ The Biot number should be as small as possible for lumped system Analysis to applicable.

→ small bodies with high thermal conductivity are good condition for lumped system analysis, especially when they are in a medium that is poor conductor of heat (such as air (or) another gas) and motion less.

→ Lumped system is valid for $Bi < 0.1$.

$$\therefore \boxed{Bi = \frac{hL_c}{K_{solid}}}$$

L_c = characteristic Length

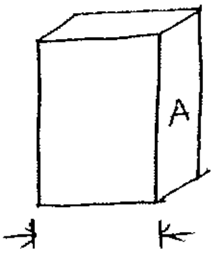
$$\boxed{L_c = \frac{\text{Volume of body}}{\text{Surface area of body.}} = \frac{V}{A}}$$

Characteristic Lengths ~~Lcm~~ for Different shapes :-

1) plane slab with convection at Both sides

$$V = AL$$

$$A_s = A + A = 2A$$



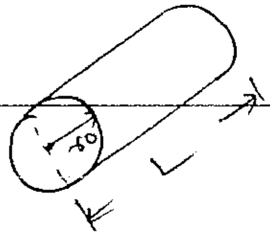
$$L_c = \frac{V}{A_s} = \frac{\cancel{A}L}{2\cancel{A}} = L/2$$

$$\therefore \boxed{L_c = L/2}$$

2) For solid cylinder of radius $\underline{r_0}$ and length $\underline{L}$ (short cylinder)
Both sides convection

$$V = \pi r_0^2 L$$

$$A = 2\pi r_0^2 + 2\pi r_0 L$$



$$L_c = \frac{\pi r_0^2 L}{2\pi r_0^2 + 2\pi r_0 L}$$

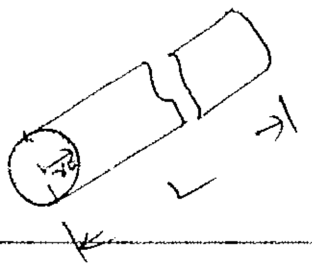
$$= \frac{\cancel{\pi} r_0^2 L}{2\cancel{\pi} r_0 (r_0 + L)}$$

$$\therefore \boxed{L_c = \frac{r_0 L}{2(r_0 + L)}}$$

3) For Long cylinder of radius r_0 ($L >> r_0$)

$$V = \pi r_0^2 L$$

$$A = 2\pi r_0 L$$

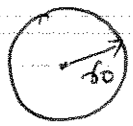


$$L_c = \frac{\cancel{\pi} r_0^2 \cancel{L}}{2\cancel{\pi} r_0 \cancel{L}} = \frac{r_0}{2} = D/4$$

$$\therefore \boxed{L_c = \frac{r_0}{2} = \frac{D}{4}}$$

4) For solid sphere of radius r_0

$$V = \frac{4}{3}\pi r_0^3$$



$$A = 4\pi r_0^2$$

$$L_c = \frac{\frac{4}{3}\pi r_0^3}{4\pi r_0^2} = \frac{r_0}{3}.$$

$$\boxed{L_c = \frac{r_0}{3} = \frac{D}{6}}$$

5) For a cube of a side L.

$$V = L^3$$

$$A = 6L^2$$

$$L_c = \frac{L^3}{6L^2} = \frac{L}{6}$$

$$\therefore \boxed{L_c = \frac{L}{6}}$$

Fourier Number :-

→ The degree of penetration of heating (or) cooling effect through the solid is called Fourier Number [Fo].

$$\rightarrow \frac{hA}{\rho C V}t = \frac{h}{\rho C L_c}t = \frac{ht}{\rho C L_c} \times \frac{L_c}{L_c} \times \frac{K}{K} \qquad \frac{hL_c}{K}$$

$$= Bi \cdot \frac{K}{\rho C} \cdot \frac{t}{L_c^2}$$

$$= Bi\, \alpha \cdot \frac{t}{L_c^2}$$

$$\boxed{\frac{hA}{\rho C V}t = Bi \cdot Fo}$$

$\therefore \boxed{\text{Fourier No} = \frac{\alpha t}{L_c^2}}$

→ It is also defined as the ratio of the rate of heat conduction to the thermal energy storage in the solid.

Conventional Questions :-

1) In a quench hardening process, steel rods ($\rho = 7832\ kg/m^3$, $c_p = 434\ J/kgK$, & $k = 63.9\ W/mK$) are heated in a furnace to 850°C and then cooled in a water bath to an average temp of 95°C. The water bath has a uniform temperature of 40°C and convection heat transfer co-efficient of $450\ W/m^2K$. If the steel rod have a diameter of 50mm and a length of 2m, determine
i) Time required to cool the steel rod from 850°C to 95°C in the water bath
ii) The total amount of heat transferred to water during the quenching of a single rod

Data

$\rho = 7832\ kg/m^3$, $c_p = 434\ J/kgK$, $k = 63.9\ W/mK$

$T_i = 850°C$, $T = 95°C$, $T_\infty = 40°C$

$h = 450\ W/m^2K$.

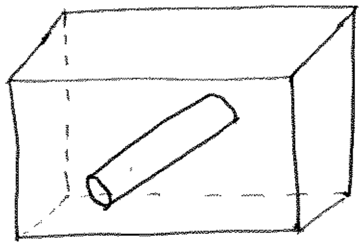
$D = 50mm = 0.05m$, $L = 2m$
$R = 0.025m$

Find:-
1) $t = ?$ to Reach 850° to 95°C.
2) Total Heat transfered during cooling

Assumptions :-

1) Thermal properties of steel rod are constant
2) convection heat transfer coefficient is uniform
3) Heat transfer by radiation is negligible

Schematic :-



sollution:-

For proceding the problem first we should find out Biot Number (Bi - non Dimensional no.) & if the Bi is less than 0.1 we can following Lumped capictance method otherwise we should follow Heisler ~~chart~~ charts

$$\therefore \; Bi = \frac{hL_c}{K}$$

$$L_c = \frac{V}{A_s} = \frac{D}{4} \text{ (or) } \frac{R}{2} =$$

$$= \frac{0.05}{4} = 0.0125m$$

$$Bi = \frac{450 \times 0.0125}{63.9} = 0.088$$

$\therefore Bi < 0.1 \rightarrow$ then we follow Lumped system Analysis

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA}{\rho C_p V}\right)t}$$

$$= e^{-\left(\frac{h}{\rho C L_c}\right)t}$$

$$\frac{95-40}{850-40} = e^{-\left(\frac{450}{7832 \times 434 \times 0.0125}\right)t}$$

$$\ln\left(\frac{95-40}{850-40}\right) = -\left(\frac{450}{7832 \times 434 \times 0.0125}\right) \times t$$

$\therefore t = 254$ sec.

2)

$$Q = mC_p(T_i - T_\infty)$$

$\rho = \frac{m}{V}$

$m = \rho \times V$.

$$= \rho V C_p (T_i - T_\infty)$$

$$= \frac{\rho \pi D^2 L C_p}{4}(T_i - T_\infty)$$

$$= \frac{7832 \times \pi \times 0.05^2 \times 2 \times 434}{4}(850 - 40)$$

$$\therefore Q = \underline{\underline{10.1 \text{ MJ}}}$$

2) An egg with mean diameter of 4cm is initially at 25°C. It is placed in boiling water for 4 min and found to be consumer's taste. For how long should be a similar egg for same consumer to boiled when taken from refrigerator at 2°C. Use lumped system Analysis and take the ~~thermo~~physical properties of egg as.

$K = 12 \, W/mK$, $h = 125 \, W/m^2K$, $C = 2000 \, J/kgK$

$\rho = 1250 \, kg/m^3$

~~Data~~

Case 1:- $D = 4cm = 0.04m$

$r_0 = 0.02m$, $T_i = 25°C$, $T_\infty = 100°C$

time$(t) = 4$ min $= 240$ sec

Case 2 :- Similar egg (geometric properties are same)

$D = 4 cm = 0.04 m$

$R_0 = 0.02 m$

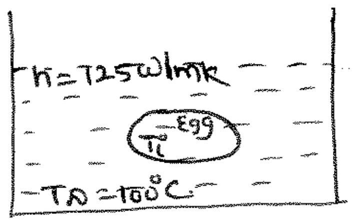
$T_i = 2^\circ C$

$k = 12 W/mK$, $h = 125 W/m^2K$, $C = 2000 J/kgK$, $\rho = 1250 kg/m^3$

Find :-

1) Time required for egg at $2^\circ C$ to be boiled to consumers taste.

Schematic :-



Assumptions :-

1) Egg as a sphere
2) constant properties
3) Boiling temp of water as $100^\circ C$

Sol :-

For proceding the problem first we should find out B_i, then proceed further processes

$$B_i = \frac{hL_c}{k}$$

$$L_c = \cancel{etc}\ \frac{V}{A_s}$$

$$= R/3 = 0.02/3 = 0.00666 m$$

$$B_i = \frac{125 \times 0.00666}{12}$$

$$B_i = 0.06944$$

$Bi < 0.1$

$\therefore$ Bi Number is less than 0.1 then lumped system Analysis is applicable

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA}{\rho C V}\right)t}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{h}{\rho C L_c}\right)t}$$

$$\frac{T - 100}{25 - 100} = e^{-\left(\frac{125}{1250 \times 2000 \times 0.02}\right) 240}$$

$$T = 87.6^\circ C$$

when egg taken ~~fr~~ from refrigerator

$T_i = 2^\circ C, \quad T = 87.6^\circ C$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{h}{\rho C L_c}\right)t}$$

$$\ln\left[\frac{87.6 - 100}{2 - 100}\right] = e^{-\left(\frac{125}{1250 \times 2000 \times 0.02}\right) t}$$

$$\therefore t = 275.6 \text{ sec.}$$

$$t = 4.6 \text{ min}$$

3) A person is found dead at 5pm in a room whose temp is ~~..~~ 20°C. The temperature of the body is measured to be 25°C, when found and Heat transfer coefficient is estimated to be 8W/m²K. Modelling the human body as 30cm diameter, 1.7m Long cylinder, calculate actual time

of death of person. take Thermophysical properties of body.

$K = 6.08\ W/mK$, $\rho = 900\ kg/m^3$, $c = 4000\ J/kgK$.

Data :-

$T_\infty = 20°C$ $\quad$ $K = 6.08\ W/mK$

$T = 25°C$. $\quad$ $\rho = 900\ kg/m^3$.

$h = 8\ W/m^2K$ $\quad$ $c = 4000\ J/kgK$.

$D = 30cm = 0.3m$

$r_0 = 0.15m$, $L = 1.7m$

Find :-

1) Actual time of death person

Assumptions :-

1) Healthy person, the body temp of 37°C at the time of death
2) No Readiation heat transfer
3) Constant properties

Sol :-

Body treated as a short cylinder

First find out B_i

$$B_i = \frac{hL_c}{K}$$

$$L_c = \frac{V}{A_s} = \frac{\pi r_0 L}{2\pi r_0 L + 2\pi r_0^2}$$

$$L_c = \frac{r_0 L}{2(r_0 + L)} = \frac{0.15 \times 1.7}{2(0.15 + 1.7)}$$

$$L_c = 0.0689\ m$$

$$\therefore B_i = \frac{8 \times 0.0689}{6.08} = 0.092$$

$\therefore B_i < 0.1$, so Lumped system is applicable

$$\frac{T-T_\infty}{T_i^\circ-T_\infty} = e^{-\left(\frac{hA}{\rho c_p V}\right)t}$$

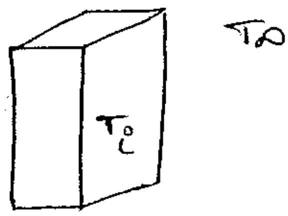
$$\frac{T-T_\infty}{T_i^\circ-T_\infty} = e^{-\left(\frac{h}{\rho c_p L_c}\right)t}$$

$$\ln\left[\frac{25-20}{37-20}\right] = -\left(\frac{8}{900\times 4000\times 0.0689}\right)t$$

$t = 87,943$ sec.

$t = 10.54$ hours before

4) In a quenching process, a copper plate 3mm thick is heated upto 400°C and then exposed to an ambient at 25°C. with the convection co-efficient of 28 W/m²K calculate the time required for the plate to reach temp of 50°C. Take Thermophysical properties as c = 380 J/kgK, ρ = 880 kg/m³, k = 38 W/mK.



Data

$L = 3\text{mm} = 3\times10^{-3}\text{m}$

$T_i^\circ = 400°C, \quad T_\infty = 25°C.$

$h = 28 W/m^2K, \quad T = 50°C$

$c = 380 J/kg, \quad \rho = 880 kg/m^3, \quad k = 38 W/mK$

Find :-

1) Time required to reach temp from 400 to 50°C.

Sol

For proceeding the problem we should find out B_i if the obtained B_i is ≤ 0.1 we should follow Numbs, if

~~Follow~~ the lumped system Analysis otherwise follow Heisler Charts ~~for~~ ~~fixed~~

$$Bi = \frac{hL_c}{K}, \quad L_c = \frac{V}{A_s} = \frac{L}{2} = 0.0015\,m$$

$$Bi = \frac{28 \times 0.0015}{38} \Rightarrow Bi = 1.0909 \times 10^{-4}$$

$\therefore$ $Bi \le 0.1 \rightarrow$ use lumped system

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{h}{\rho c L_c}\right)t}.$$

$$\ln\left[\frac{50 - 25}{400 - 25}\right] = -\left(\frac{28}{880 \times 380 \times 0.0015}\right) \times t.$$

$$\therefore\ t = 485\ sec = 8.08\ min$$

5) A Titanium alloy blade of an axial compressor for which $K = 25\,W/mK$, $\rho = 4500\,kg/m^3$, & $c = 520\,J/kgK$ is initally at $60°C$. The effective thickness of the blade is 10mm and then exposed to gas stream at $600°C$. The blade experiences a heat transfer coefficient of $500\,W/m^2K$, use low Biot number approximation to estimate temp of blade after 1, 5, 20 & 100sec

<u>Data</u> $K = 25\,W/mK$, $\rho = 4500\,kg/m^3$, $c = 520\,J/kgK$

$T_i = 60°C$, $L = 10mm = 10 \times 10^{-3}\,m$, $T_\infty = 600°C$.

$h = 500\,W/m^2K$,

Find:-

1) Temparatures at 1, 5, 20 & 100 sec

sol

$Bi = \frac{hL_c}{k}$ $L_c = \frac{10\times10^{-3}}{2} = 5\times10^{-3}$ m.

∴ Low Bi number approximation is

$$\frac{T-T_\infty}{T_i-T_\infty} = e^{-\left(\frac{h}{\rho c L_c}\right)t}$$

$$\frac{T-600}{60-600} = e^{-\left(\frac{500}{4500\times520\times5\times10^{-3}}\right)\times 1}$$

$T = 82.6°C$.

and Further do same procedure for ~~5, 20 & 100 sec~~ 5, 20, & 100 sec

6) A Thermocouple junction, which may be approximated as a sphere; It is to be used for temp measurement in a gas stream. The convection b/w the junction surface and the gas $h = 400\ W/m^2K$ and junction Thermophysical properties are $k = 20 W/mK$, $C = 400\ J/kgK$, $\rho = 8500\ kg/m^3$. Determine the junction diameter needed for the thermocouples to have a time constant of 1 sec. If the thermocouples junction is at 25°C and is placed in a gas stream that is at 200°C, how long it will take for the junction to reach 199°C.

Data

$h = 400\,W/m^2K$, $K = 20\,W/mK$, $c = 400\,J/kgK$, $\rho = 8500\,kg/m^3$

$T_i = 25^\circ C$, $T_\infty = 200^\circ C$, $T = 199^\circ C$

~~Sol~~ Find

1) Dia of Thermocouple $(d) = ?$

2) Time required to reach temp ~~of~~ From $25^\circ C$ to $199^\circ C$.

Sol

According Time Repsome and Response of Thermocouples

$$\boxed{t = \frac{\rho V C}{h A_s}}$$

$$t = \frac{\rho c L_c}{h}$$

$$1 = \frac{8500 \times 400 \times L_c}{400}$$

$$\Rightarrow L_c = 1.176 \times 10^{-4}\,m$$

$\therefore L_c = \frac{V}{A}$ Thermo couples is a sphere

$\therefore$ for sphere $L_c = \frac{D}{6} \Rightarrow D = 7.058 \times 10^{-4}\,m$

$\therefore$ further proceeding is

$$Bi = \frac{hL_c}{k} \Rightarrow Bi = \frac{400 \times 7.058 \times 10^{-4}}{20} = 0.014$$

$\therefore Bi \le 0.1 \rightarrow$ lumped system Analysis should follow.

$$\ln\left|\frac{T - T_\infty}{T_i - T_\infty}\right| = e^{-\left(\frac{h}{\rho c L_c}\right)t} \Rightarrow \ln\left|\frac{199 - 200}{25 - 200}\right| = -\frac{400}{8500 \times 400 \times 7.058 \times 10^{-4}} \times t$$

$$\therefore t = 5.2\,sec$$

Time Constant ~~&~~ and Response of a Thermocouple :-

→ When two dissimilar metals are joined together at two points to form a couple loop and a temperature difference exists b/w the junctions, an electrical potential is setup b/w the junction.

→ Such Arrangement is known as Thermocouple and used for the measuring of temperature.

→ Measurement of temperature by thermocouple is an important application of the lumped system ~~and~~ analysis

→ The ~~Response~~ ~~Rep~~ "Response of a thermocouple" is defined as the time required for the thermocouple to reach the source temperature when it is exposed to it.

→ The Time constant Thermocouple is Rearranged from lumped parameter & it is finally

$$\frac{T-T_\infty}{T_i-T_\infty} = e^{-\left(\frac{hA}{\rho VC}\right)t}$$

→ The sensitivity of the Thermocouple is defined as the time required by the thermocouple to reach 63.2% of its steady state value.

$$\frac{T-T_\infty}{T_i-T_\infty} = e^{\left(\frac{-hA}{\rho VC}\right)t} = 1 - \cancel{0.632} = 0.368$$

$$= e^{-1}$$

$$\cancel{1-0.632} = \cancel{e^{\left(\frac{hA}{\rho VC}\right)t}}$$

$$\cancel{1-0.632} = \cancel{e^{-\left(\frac{hA}{\rho CV}\right)t}}$$

$$\cancel{0.368}$$

$$e^{(-1)} = e$$

$$1 - 0.632 = e^{-\left(\frac{hA}{\rho c V}\right)t}$$

$$0.368 = e^{-\left(\frac{hA}{\rho c V}\right)t}$$

$$e^{(-1)} = e^{-\left(\frac{hA}{\rho c V}\right)t} \quad \Big| \; e^{(-1)} = 0.368$$

$$1 = \frac{hA}{\rho c V} t$$

$$\therefore \boxed{t = \frac{\rho V c}{hA} = \frac{\rho c}{h L_c}}$$

→ The time constant is indicative of the speed of response
ie 1) ↑ t ⇒ slow of the response time
2) ↓ t ⇒ high of the response time

→ so as we are engineers to develop a thermocouple to response fastly and required low value of time & it can be achieved by.

1) Decreasing the diameter of wire
2) using light materials of low density and low specific heat [Mean less Heat storage capacity]
3) Increasing the heat transfer co-efficient.

→ Depending upon the type of fluid used, the response times for different sizes and materials of Thermocouple wires usually lie b/w. 0.04 to 2.5 seconds

Transient Temperature Charts :- [HEISLER & GROBER CHARTS]

→ one the Bi number is known, ~~and~~ ~~the~~ ~~DB~~ if we got Low Bi number we should not use lumped capicitance to find out time (or) required temp of Heat etc - - .

→ ~~For~~ when the internal temperature gradients are Large, Lumped heat capacitance system becomes unsuitable for the Analysis of Transient heat conduction problems.

→ ~~In~~ ~~S~~ ~~The~~ In such situation the Heisler and grober charts are widely used

→ The Transient temperature charts for a large plane wall, Long cylinder and sphere were represented by M.P. Heisler in 1947 and are called Heisler charts. They were supplemented in 1961 with heat transfer charts by H. Grober.

→ There are Three charts assosiated with each Geometry.

1) The First chart is used for to determine the temperature To at the Centre of the geometry at given time t.

2) The second chart is used for to determine the temperature at other Locations at the same time in terms of To.

3) The Third chart is to determine the total amount of heat transfer up to the time t.

→ Therefore The Heisler Charts are widely used for determine

1) centre line temparature
2) position temparature
3) The heat transfer

→ To obtain the required value of unknowns, the various dimensionless parameters required are

~~Temparature~~

1) Biot number [B_i]
2) Fourier number [F_o]
3) Temparature ratio at the centre [θ_c/θ_i]
4) Temparature ratio at any position [θ/θ_c]
5) Dimensionless position [x/L]
6) Dimensionless Heat transfer [Q/Q_i]

1) Transient Temparature Charts for slab :-

→ Consider a slab (ie plane) of thickness $2L$, confined to the region $-L \leq x \leq L$. The slab initially at temp T_i, is suddenly exposed to convection environment (for $t > 0$) with a heat transfer coefficient h, on its both boundary surfaces.

→ The heat flow from both surfaces in-ward, Due to symmetry of problems, only half region $0 \leq x \leq L$ is considered

→ The Dimensionless parameters for slab can be expressed as

1) Biot Number [B_i] $= \dfrac{hL}{k}$

2) Fourier Number [F_o] $= \dfrac{\alpha t}{L_c^2}$

3) Temparature ratio at the centre $\left[\frac{\theta_c}{\theta_i}\right] = \frac{T_0 - T_\infty}{T_i - T_\infty}$

4) Temparature ratio at any position $\left[\frac{\theta}{\theta_c}\right] = \frac{T(x,t) - T_\infty}{T_0 - T_\infty}$

5) Dimension Less position $= \frac{x}{L}$

6) Dimension Less heat Transfer $= \frac{Q}{Q_i}$

where

L = half thickness of a slab — m

x = position in the slab measured from from centre where the temp is required — m

T_0 = centre line temp in the slab — °C

$T(x,t)$ = position temp in the slab — °C

T_i = Initial temp in the slab — °C

t = time — seconds

Q = total amount of energy lost by plate during time t — J

Q_i = inital internal energy content in the slab

$= mc_p \Delta T$ —

$= \rho V c_p \Delta T$

$= \rho (A\,2L) c_p (T_i - T_\infty)$ — Joules

→ The temp at any position x from the Mid-plane can be obtained from position correction temparature chart

$$\frac{\theta}{\theta_i} = \frac{\theta_c}{\theta_i} \times \frac{\theta}{\theta_c} = \frac{T - T_\infty}{T_i - T_\infty}$$

~~2) Temperature Transient Charts~~ for

2) Transient Temperature Charts for Long cylinder & sphere:-

→ consider a long cylinder or a sphere of radius r_0, initally at temp T_i is suddenly subjected to convection envirorment for $(t>0)$ with heat transfer co-efficient h and fluid temp T_∞, is

1) Biot number $[B_i] = \frac{hr_0}{k}$

2) Fourier number $[F_0] = \frac{\alpha t}{r_0^2}$

3) Temp ratio at the centre $\left[\frac{\theta_0}{\theta_i}\right] = \frac{T_0 - T_\infty}{T_i - T_\infty}$

4) Temp ratio at any position $\left[\frac{\theta}{\theta_0}\right] = \frac{T(r,t) - T_\infty}{T_i - T_\infty}$

5) Dimension less radial position $= \frac{r}{r_0}$

6) Dimension less heat transfer $= \frac{Q}{Q_i}$

where

r_0 = radius of cylinder (or) sphere – m

r = position radius in cylinder (or) sphere – m

$T(r,t)$ = position temp – °C

α = Thermal diffusivity – m^2/s

K = Thermal conductivity = W/mK

h = heat transfer coefficient = W/m^2K

Conventional questions :-

1) A 50mm thick iron plate is initially at 225°C. Its both surfaces are suddenly exposed to an Environment at 25°C. with convection co-efficient of 500W/m²K. calculate

1) The centre temperature, 2minutes after the state of Exposure

2) The temp at the depth of 10mm from the surface after 2min of exposure

3) Energy removed from the plate per square meter during this period. Take Thermophysical properties of $k = 60\ W/mK$, $\rho = 7850\ kg/m^3$, $c = 460\ J/kgK$, $\alpha = 1.6\times10^{-5}\ m^2/s$.

Data

$2L = 50mm \Rightarrow L_c = 25mm = 0.025m$

$T_i = 225°C$, $T_\infty = 25°C$, $h = 500\ W/m^2K$

$t = 2min = 120sec.$

Find:-

1) $T_{centre} = ?$

2) $T_{x/L} = ?$

3) $Q = ?$

Assumption:-

Sol :-

Considering the plate thickness 2L, hence consider L_c as characteristic length (2L/2)

For doing transient problems, first we should find out B_i, if B_i is less than 0.1 follow lumped capacitance otherwise use Heisler charts

$$B_i = \frac{hL_c}{K} = \frac{500 \times 0.025}{60} = 0.21$$

$\therefore B_i > 0.1$ use Heisler charts for further proceedings

$$F_0 = \frac{\alpha t}{L_c^2} = \frac{1.6 \times 10^{-5} \times 120}{(0.025)^2} = 3.07.$$

1) From Heisler chart of plane slab for centre line temperature from data book

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty}$$

for $B_i = 0.21$ & $F_0 = 3.07$ The $\frac{\theta_c}{\theta_i}$ is

$$\frac{\theta_c}{\theta_i} = 0.58 \Rightarrow 0.58 = \frac{T_0 - 25}{225 - 25}$$

$$\therefore T_0 = 141^\circ C$$

2) Temp at depth of 10 mm from the surface (x/L)

$$x = L_c - \text{depth}$$
$$= 25 - 10 = 15\,mm$$
$$\therefore \frac{x}{L} = \frac{15}{25} = 0.6.$$

from data book $B_i = 0.21$ & $x/L = 0.6$

$$\frac{\theta}{\theta_c} = 0.95$$

$$\frac{\theta}{\theta_c} \cancel{0.95} = \frac{T_{x/L} - T_\infty}{T_0 - T_\infty}$$

$$0.95 = \frac{T_{x/L} - 25}{141 - 25} \Rightarrow T_{x/L} = 135.2^\circ C$$

3) Heat Loss from the plate during 2 sec

From data book B_i & $B_i^2 F_o$ Charts

$$B_i = 0.21$$
$$B_i^2 F_o = 0.21^2 \times 3.07$$
$$= 0.135$$

$$\therefore \frac{Q}{Q_i} = 0.45 \Rightarrow Q = Q_i \times 0.45$$

$$\therefore Q_i = m c_p dT$$
$$= \rho V c_p dT$$
$$= \rho(A \times 2 \times L_c) \times c_p (T_i - T_\infty)$$

$$= 7850\,(\cancel{0.1}\ 2\times 0.25) \times 460\,(225-25)$$

$$= 35.33\times 10^3 \ kJ/m^2$$

$$\therefore q = 0.45\times 35.33\times 10^3$$

$$= 15.9\times 10^3 \ kJ/m^2$$

2) The nose ~~of~~ section of a missile is formed of a 6mm thick stainless steel plate and is held initially at uniform temp of 88°C. The missile enters the denser Layers of the atmosphere at a very high velocity. The effective temp of air surrounding the nose region attains the value 2200°C. and the surface convective co-efficient is estimated at 3405 w/m²K. Make calculations for the max permissible time in these surroundings if the max metal temp is not to exceed 1095°C. Also work ~~or~~ out the inside surface temp under these conditions. The constant values for steel properties are, $\rho = 7850$ kg/m³, $c_p = 465$ J/kgK, $k = 54$ w/mk

Data

$2L = 6mm \Rightarrow L_c = 6/2 = 3mm = 3\times 10^{-3}\ m$

$T_i = 88°C$, $T_\infty = 2200°C$, $h = 3405\ w/m^2k$, $T_0 = 1095°C$ (find t)

$\rho = 7800\ kg/m^3$, $c = 465\ J/kgK$, $k = 54\ w/mk$

Find:-

1) $t = ?$ for reaching 1095°C at outside surface ($x = L_c$)

2) $T_0 = ?$ at inside surface ($x = 0$)

Assumptions :-

Sol

i) $Bi = \frac{h L_c}{K} = \frac{3405 \times 3 \times 10^{-3}}{54} = 0.189$, $Bi > 0.1$

$F_o = \frac{\alpha t}{L_c^2} = \frac{K}{\rho c_p} \times \frac{t}{L_c^2}$, we shud find out 't'

$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{1095 - 2200}{88 - 2200} \simeq 0.5232$

1) from data book of Heisler charts of $\frac{\theta}{\theta_c} = 0.5232$ &

$Bi = 0.189$.

$F_o = 3.6$

$\therefore F_o = \frac{k}{\rho c_p} \times \frac{t}{L_c^2} \Rightarrow 3.6 = \frac{54 \times t}{7800 \times 465 \times (3 \times 10^{-3})^2}$

$\therefore t = \underline{2.17 \text{ sec}}$

2) Inside temp (take at $x=0$)

$\frac{\theta_c}{\theta_i} = 0.5232$

$\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.5232 \Rightarrow \frac{T_0 - 2200}{88 - 2200} = 0.5232$

$\therefore T_0 = 1095^\circ C$

3) A long ~~cylinder~~ Aluminium cylinder 5cm in diameter and initially at 200°C is suddenly exposed to convection environment at 70°C. with heat transfer co-efficient of 525 W/m²K. calculate the temp at the radius of 1.25cm 1minute after the cylinder is exposed to the environment. Take $\rho = 2700\ kg/m^3$, $c = 900\ J/kgK$, $k = 210\ W/m^2K$

Data

$d_0 = 5cm$, $r_0 = 2.5cm = 0.025m$, $t = 1min = 60sec$

$T_i = 200°C$, $T_\infty = 70°C$, $h = 525\ W/m^2K$

$\rho = 2700\ kg/m^3$

Find:-

1) $T_{x/L} = ?$

Assumptions:-

Sol

First find out centre line Temp from Heisler Chart

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty}$$

$$B_i = \frac{hL_c}{K} = \frac{525 \times 0.025}{210} = 0.0625$$

$$F_0 = \frac{\alpha t}{L_c^2} \Rightarrow \frac{k}{\rho c_p} \times \frac{t}{L_c^2} \Rightarrow \frac{210 \times 60}{2700 \times 900 \times 0.025^2}$$

$F_0 = 8.29$

From Heisler chart with F_0 & B_i

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.2$$

$$\frac{T_0 - 70}{200 - 70} = 0.2 \Rightarrow T_0 = 96^\circ C$$

Temp at any position $(r/r_0) = \frac{1.25}{2.5} = 0.5$

$\therefore$ r/r_0 & B_i charts

$$\frac{\theta}{\theta_c} = \frac{T_{(r/r_0)} - T_\infty}{T_0 - T_\infty} \Rightarrow 0.78 = \frac{T_{(r/r_0)} - 70}{96 - 70}$$

$$\therefore T_{(r/r_0)} = 90.28^\circ C$$

4) A long cylinder shaft 20cm in diameter is made of steel ($k = 14.9\ W/mk$), $\rho = 7900\ kg/m^3$, $C = 477\ J/kgk$ and $\alpha = 3.95 \times 10^{-6}\ m^2/s$. It comes out an oven at a uniform temp of $600^\circ C$. The shaft is then allowed to cool slowly in an environment at $200^\circ C$ with convective heat transfer co-efficient of $80\ W/m^2k$. calculate the temp at the centre of the cylinder ~~after~~ 5-minutes after the start of cooling process. Also calculate the heat transferred per unit length of the shaft during this period.

Data

$d = 20cm$, $r_0 = 10cm = 0.1m$, $k = 14.9\ w/mk$

$\rho = 7900\ kg/m^3$, $c = 477\ J/kgk$, $\alpha = 3.95 \times 10^{-6}\ m^2/s$

$T_i = 600°c$, $T_\infty = 200°c$, $h = 80\ w/m^2k$, $t = 45\ min$
$= 2700\ sec.$

Find:-

1) $T_0 = ?$ at centre of the shaft after 45 min

2) Heat transfer $(Q) = ?$

Assumptions:-

Sol:-

$$B_i = \frac{hr_0}{k} = \frac{80 \times 0.1}{14.9} = 0.537$$

$$F_0 = \frac{\alpha t}{r_0^2} \Rightarrow F_0 = \frac{3.95 \times 10^{-6} \times 2700}{(0.1)^2} = 1.0665$$

From data book centreline temp charts with B_i & F_0

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.5$$

$$\frac{T_0 - 200}{600 - 200} = 0.5 \Rightarrow T_0 = 400°c.$$

From data book Heat flow chart with B_i & $B_i^2 F_0$

$$Bi^2 Fo = (0.537)^2 \times 1.0665$$

$$= 0.3075$$

$$\therefore \frac{Q}{Q_i} = 0.62 \Rightarrow Q = 0.62 \times Q_i$$

$$Q_i = m c_p dT \Rightarrow \rho V c_p dT \Rightarrow \rho \times \frac{\pi}{4} r_0^2 L \times c_p (T_0 - T_\infty)$$

$$\therefore Q = 0.62 \times 7900 \times \frac{\pi}{4} \times (0.1)^2 \times 1 \times 477 (600-200)$$

$$\therefore Q = \underline{\underline{29.36 \text{ MJ}}}$$

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5) An egg can be approximated as a sphere, 5cm in dia with Thermophysical properties $K = 0.6 \, W/mK$, $\alpha = 0.14 \times 10^{-6} \, m^2/s$. The egg is taken from a refrigerator at 2°C and is dropped into boiling water, where the convection heat transfer coefficient estimated as $1200 \, W/m^2K$. calculate the time required to reach the centre temp of the egg to 75°C.

Data

$d = 5cm, \Rightarrow r_0 = 2.5cm = 0.025m$, $k = 0.6 \, W/mK$

$\alpha = 0.14 \times 10^{-6} \, m^2/s$, $T_i = 2°C$, $T_\infty = 100°C$ (Dropped in Boiling water), $h = 1200 \, W/m^2K$, $T_0 = 75°C$.

Find:-

1) $t = ?$ for reaching 75°C

Assumptions:-

Sol

$$B_i = \frac{h r_0}{K} = \frac{1200 \times 0.025}{0.6} = 50.$$

$Fo = \frac{\alpha t}{r_0^2}$ => Here we should find out 't'

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{75-100}{2-100} = 0.255$$

with $\frac{\theta_c}{\theta_i}$ & B_i.

we donot have any chart with $B_i = 50$

so we can use ~~for~~ High Biot number ie, one Term sollution

$$\boxed{\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 e^{-\xi^2 Fo}}$$

↳ one Term sollution

with sphere $B_i = 50$ from tables

$$\xi = 3.0788 \quad \& \quad C = 1.9962$$

$$\therefore \frac{75-100}{2-100} = 1.9962\, e^{-(3.0788)^2 \times Fo}$$

$$\therefore Fo = 0.2154 \Rightarrow Fo = \frac{\alpha t}{L_c^2}$$

$$0.1254 = \frac{0.14 \times 10^{-6} \times t}{(0.025)^2}$$

$$\therefore t = 961.6\,sec \Rightarrow t = 16\,min$$

6) A 3.6 cm diameter egg, approximated spherical in shape, is initially at 25°C temp. To boil it to the consumer's taste, it needs to be placed 225 seconds in a saucepan of boiling water at 100°C. For how long should a similar egg for the same consumer be boiled of egg taken from a refrigerator at temp 5°C. Thermo physical properties of egg is $K = 2.5 \, W/mK$, $\rho = 1250 \, kg/m^3$, $c = 2200 \, J/kgK$, $h = 2800 \, W/m^2K$.

Data

$d = 3.6 \, cm$, $r_0 = 0.18 \, cm = 0.18 \times 10^{-2} \, m$

$T_i = 25°C$, $T_\infty = 100°C$, $t = 225 \, sec.$

For similar taken from refrigerator

$T_i = 5°C$, $T_\infty = 100°C$.

$K = \cancel{2.5}$.

Find:-

1) For time @ $t = 225 \, sec$ $T_0 = ?$ at $T_i = 25°C$

2) For T_0 time $t = ?$ at $T_i = 5°C$.

Assumptions:-

Sol

$$B_i = \frac{h r_0}{K} = \frac{2800 \times 0.18 \times 10^{-2}}{2.5} \Rightarrow B_i$$

$$\therefore B_i = 2.016$$

$$Fo = \frac{\alpha t}{L_c^2} \Rightarrow Fo = \frac{K}{\rho c_p} \frac{t}{L_c^2}$$

$$Fo = \frac{2.5 \times 225}{1250 \times 2200 \times (0.18 \times 10^{-2})^2}$$

$$Fo = 63.13$$

From data book centre line temp [because in sphere the HT towards centre] of Heisler charts with Bi & Fo

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.2$$

$$\frac{T_0 - 100}{25 - 100} = 0.2$$

$$\therefore T_0 = 83.5°C$$

For similar egg at 5°C to reach T_0 =

$$\frac{T_0 - T_\infty}{T_i - T_\infty} \Rightarrow \frac{83.5 - 100}{5 - 100} = 0.174$$

$\therefore$ from $\frac{\theta_c}{\theta_i}$ & Bi ∞ in Heisler chart

~~Fo~~

$$Fo = 0.71$$

$\therefore$

$$\therefore F_o = \frac{\alpha t}{L_c^2}$$

$$= \frac{K}{\rho c_p} \frac{t}{L_c^2}$$

$$= \frac{2.5 \times t}{1250 \times 2200 \times (0.18 \times 10^{-2})^2}$$

$$\therefore t = \underline{\underline{253 \text{ seconds}}}$$